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GRIBOV'S THEOREM ON SOFT EMISSION AND THE REGGEON-REGGEON-GLUON VERTEX AT SMALL TRANSVERSE MOMENTUM *

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Abstract

In spite of the fact that the Gribov's theorem about the region of applicability of the soft emission factorization cannot be referred literally to the case of massless charged particles, it can be used for the calculation of soft emission amplitudes in such a case also. We demonstrate this for the gluon production amplitude in the multi-Regge kinematics with small transverse momentum.

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In 1967 V.N. Gribov showed [1] that at high energy hadron collisions the region of applicability of well-known formulas for accompanying bremstrahlung extends considerably. Namely, for collision of two particles, A and B, with large c.m.s. energy $\sqrt{s} = \sqrt{(p_A + p_B)^2}$ this region is restricted by the inequalities

$$\frac{2p_A k}{s} \ll 1 \ , \qquad\qquad \frac{2p_B k}{s} \ll 1 \ ,$$

$$\vec{k}_\perp^{\,2} \approx \frac{2(p_A k) \cdot 2(p_B k)}{s} \ll \mu^2 \ , \tag{1}$$

where $\vec{k}_\perp$ is the projection of the momentum of the emitted photon on the plane orthogonal to the momenta of the colliding particles p_A and p_B, and μ is a typical hadron mass.

Gribov proved [1] that in the region (1) the amplitude of the emission process is given only by those Feynman diagrams where the photon line is attached to external charged particles. Furthermore, calculating the contributions of these diagrams one has to keep the non radiative part of the amplitude on the mass shell, i.e. to neglect virtualities of radiating particles. In the following we shall call the formulas obtained in such a way “soft insertion formulas”. Let us stress that these formulas are invariant under gauge transformation of the emitted photon.

Notice that before the work of Ref. [1] it was generally accepted (see, for example, Ref. [2]) that for the applicability of the soft insertion formulas one has to have

$$2p_A k \ll \mu^2 \ , \qquad\qquad 2p_B k \ll \mu^2 \ . \tag{2}$$

Indeed, the conditions (2) are much more stringent than the conditions (1). The possibility of using the factorized formulas with the on mass-shell non radiative amplitude in the region (1) is quite non trivial and is connected with gauge invariance of the emission amplitude [1].

It is very attractive to make use of the Gribov’s theorem in more complicated cases, such as, for example, Quantum Chromodynamics (QCD). An evident obstacle

for this is the masslessness of particles having colour charge. In other words, the typical mass μ in Eq. (1) is equal to zero for the case of QCD.

The main point in the proof of the Gribov's theorem is the smallness of the transverse momentum $k_\perp$ of the emitted quantum of the gauge field (photon or gluon) in comparison with the essential transverse momenta of the other particles. In massive theories the latter momenta are of order (or larger than) μ. Contrary, in theories with massless particles, such as QCD, the essential transverse momenta of virtual particles can be arbitrary small (that appears as infrared and collinear divergences). Therefore, the Gribov's theorem cannot be applied literally for these theories.

Nevertheless, the theorem can be used for the calculation of emission amplitudes. Below we demonstrate this for the process of emission of a gluon G with momentum $p_G \equiv k$ at scattering of the particles (quarks or gluons) A and B,

$$A + B \to A' + B' + G \ , \tag{3}$$

in the multi-Regge kinematics

$$\begin{gathered} s = (p_A + p_B)^2 \gg s_{1,2} \gg |t_{1,2}| \ , \\ s_1 = (2p_{A'} + k)^2 \approx 2p_A k \ , \qquad s_2 = (2p_{B'} + k)^2 \approx 2p_B k \ , \\ t_i = q_i^2 \approx -\vec{q}_{i\perp}^{\,2} \ , \qquad q_1 = p_A - p_{A'} \ , \qquad q_2 = p_{B'} - p_B \ , \end{gathered} \tag{4}$$

for the case of the transverse momentum of the emitted gluon small compared with the transferred momenta:

$$\begin{gathered} |\vec{k}_\perp| \ll |\vec{q}_\perp| \ , \qquad q \equiv \frac{q_1 + q_2}{2} \ , \\ |t_1 - t_2| \ll |t| \approx \vec{q}_\perp^{\,2} \ . \end{gathered} \tag{5}$$

This process is chosen because of its connection with the small x behaviour of parton distributions (x is the fraction of the hadron momentum carried by a parton). In the leading $ln(1/x)$ approximation (LLA) these distributions can be calculated using the BFKL equation [3]. To define a region of its applicability as well as to fix a scale of virtualities of the running coupling constant $\alpha_s(Q^2)$ in the equation one has to know radiative corrections to the kernel of the equation. The program of calculation of the next-to-leading corrections was developed in Ref. [4]. It is based on the gluon Reggeization in QCD; therefore, the corrections to the kernel include the one-loop corrections to the Reggeon-Reggeon-gluon (RRG) vertex, which are determined by the gluon production amplitude in the multi-Regge kinematics. The corrections to the RRG vertex were calculated in Refs. [5,6]. The calculations were performed in the space-time dimension $D \neq 4$ for regularizing the infrared and collinear divergences, but terms vanishing at $D \to 4$ were omitted in the final expressions. Unfortunately, such terms can give non vanishing contributions to the total cross sections (and to corrections to the kernel of the BFKL equation) because the integration over the transverse momenta of the produced gluon leads to divergences at $k_\perp = 0$ for the case $D = 4$. Therefore, in the region $k_\perp \to 0$ we need to know the production amplitude for arbitrary $D \neq 4$. As we shall see, the use of the Gribov's theorem simplifies considerably the calculation of the amplitude in this region.

Let us start with the Born approximation. Obviously, the soft insertion formula should be valid here in the region defined by Eqs. (4) and (5), because all transverse momenta are fixed and $k_\perp$ is the smallest one. The elastic scattering amplitude in the region of large s and fixed t in the Born approximation has the form

$$\mathcal{A}_{AB}^{A'B'}(Born) = \Gamma_{A'A}^{(0)i} \frac{2s}{t} \Gamma_{B'B}^{(0)i} \ , \tag{6}$$

where $t = -\vec{q}_\perp^{\,2}$ and $\Gamma^{(0)i}_{A'A}$ are the particle-particle-Reggeon (PPR) vertices in the Born approximation [3]. In the helicity basis these vertices can be presented as

$$\Gamma^{(0)i}_{A'A} = g\langle A'|T^i|A\rangle \delta_{\lambda_{A'},\lambda_A} \ , \tag{7}$$

where $\langle A'|T^i|A\rangle$ are the matrix elements of the colour group generators in the corresponding representation. It is easy to see that the soft insertion of a gluon with momentum $p_G \equiv k$, colour index c and polarization vector $e(k)$ gives us

$$\mathcal{A}^{A'GB'}_{AB}(Born) = \Gamma^{(0)i_1}_{A'A}\frac{2s}{t}\Gamma^{(0)i_2}_{B'B}gT^c_{i_2 i_1}e^*_\mu(k)\left(\frac{p_A^\mu}{p_A k} - \frac{p_B^\mu}{p_B k}\right) \ . \tag{8}$$

Let us remind that in the kinematics defined by the relations (4) the gluon production amplitude in the Born approximation takes the form [3]

$$\mathcal{A}^{A'GB'}_{AB}(Born) = 2s\Gamma^{(0)i_1}_{A'A}\frac{1}{t_1}gT^c_{i_2 i_1}e^*_\mu(k)C^\mu(q_2,q_1)\frac{1}{t_2}\Gamma^{(0)i}_{B'B} \ , \tag{9}$$

where the effective production vertex is

$$C(q_2,q_1) = -q_{1\perp} - q_{2\perp} + p_A\left(\frac{q_1^2}{p_A k} + \frac{p_B k}{p_A p_B}\right) - p_B\left(\frac{q_2^2}{p_B k} + \frac{p_A k}{p_A p_B}\right) \ . \tag{10}$$

In the region (5) of small $k_\perp$ we find that

$$C(q_2,q_1) \to t\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right) \ , \tag{11}$$

and the expression (9) turns into the form (8). So, for the case of the Born approximation the soft insertion formula is valid, as it was reported.

Now let us consider the one-loop corrections to the production amplitude. Since in this case we need to integrate over the momenta of virtual particles, we cannot expect that the soft insertion gives a corrected answer here. However, analyzing the proof of the Gribov's theorem [1] one can conclude that the soft insertion should be valid for the contribution of the kinematical region where the transverse momenta

of virtual particles are much larger than $k_\perp$. The idea is to use the soft insertion formula for this contribution and to add the contribution of the region of small virtual transverse momenta, which has to be calculated separately. From the first sight the idea appears doubtful, because for $D = 4$ the integrals over virtual transverse momenta have a logarithmic behaviour; therefore, it seems that the separation of two regions is not a simple problem. But for $D > 4$ the integrals are convergent, and we have two different scales where they can converge, $q_\perp$ and $k_\perp$, so that the separation is quite simple in this case. Evidently, the contribution of the integrals converging at $q_\perp$ can be obtained applying the soft insertion formula and the contribution of the integrals converging at $k_\perp$ has to be calculated.

Fortunately, a simple inspection of the Feynman diagrams shows that only those ones of Fig. 1 lead to the integrals of the second kind. This statement is valid for all possible choices of colliding particles: they can be gluons (in this case all lines in the diagrams of Fig. 1 are gluon lines) or quarks (in this case the upper and lower lines in the diagrams are quark lines) and so on.

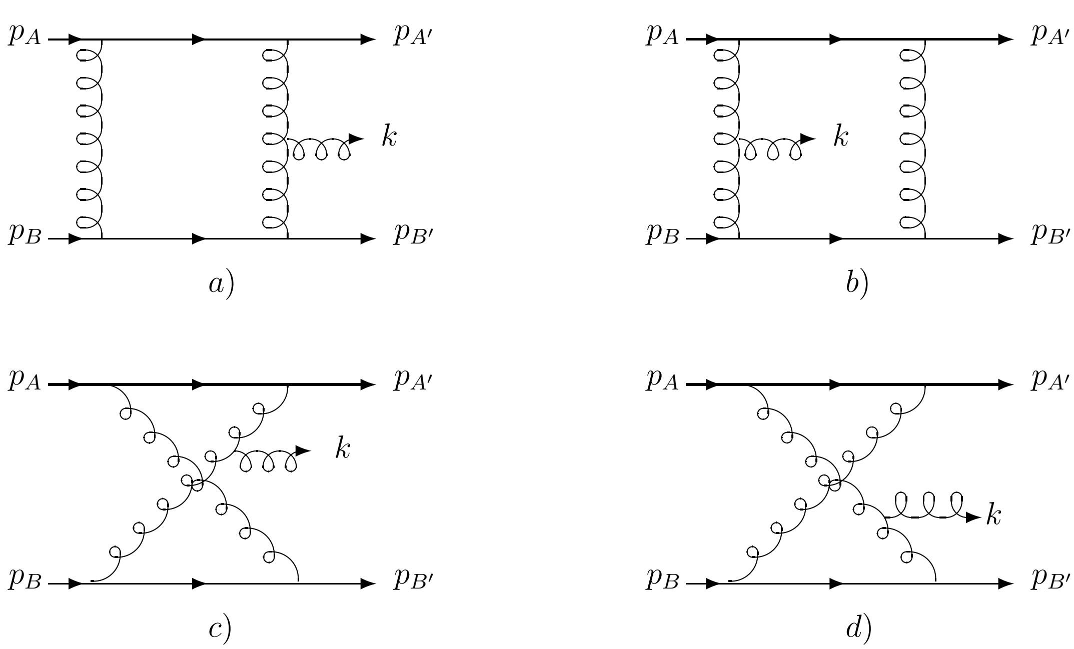

Fig. 1

Fig. 1: Feynman diagrams giving a non factorizable contribution to the gluon emission amplitude.

Let us split the production amplitude as the sum of the factorizable and non contributions:

$$\mathcal{A}_{AB}^{A'GB'} = \mathcal{A}_{AB}^{A'GB'}(f) + \mathcal{A}_{AB}^{A'GB'}(nf) \ . \tag{12}$$

The first term in Eq. (12) comes from the soft insertion while the second one is represented in the one-loop approximation by the diagrams of Fig. 1.

Contrary to the Born case, in higher orders the colour structure of the production amplitude is not so simple. For definiteness, let us consider the part of the amplitude with the gluon quantum numbers in t_1 and t_2 channels. This part is the most important one because it determines the RRG vertex. The factorizable contribution to this part has a form similar to the expression (8):

$$\mathcal{A}_{AB}^{(8)\ A'GB'}(f) = \Gamma_{A'A}^{i_1}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma_{B'B}^{i_2} g T_{i_2 i_1}^{c} e_\mu^*(k)\left(\frac{p_A^\mu}{p_A k} - \frac{p_B^\mu}{p_B k}\right) \ . \tag{13}$$

Here $j(t) = 1 + \omega(t)$ is the gluon trajectory [3] and $\Gamma_{A'A}^{i}$ are the PPR vertices. The one-loop corrections to the LLA vertices (7) are calculated in Refs. [5,7]. There one can find an explicit expression for $\omega(t)$ also.

Now let us pass to the non factorizable contribution. Evidently, the diagrams of Fig. 1 are connected each other by crossing, therefore it is sufficient to calculate the contribution of the diagram a). Performing usual tricks with the numerators of the gluon propagators connecting lines with strongly different momenta:

$$g^{\mu\nu} \to \frac{2p_A^\mu p_B^\nu}{s} \ , \tag{14}$$

and simplifying the numerators of the integrand allow to present the contribution of the diagram a) of Fig. 1 in the form

$$\mathcal{A}_{AB}^{(8)\ A'GB'}(a) = -\frac{g^3 N}{4}\Gamma_{A'A}^{(0)\ i_1}\Gamma_{B'B}^{(0)\ i_2} T_{i_2 i_1}^{c} e_\mu^*(k)\left(\frac{p_A^\mu}{p_A k} - \frac{p_B^\mu}{p_B k}\right) s s_1 s_2 \mathcal{I} \ , \tag{15}$$

where

$$\mathcal{I}=\int\frac{d^Dp}{(2\pi)^Di(p^2+i\varepsilon)((p+p_A)^2+i\varepsilon)((p-p_B)^2+i\varepsilon)((p+q_1)^2+i\varepsilon)((p+q_2)^2+i\varepsilon)}\,. \tag{16}$$

In the region defined by the relations (4) and (5) we get

$$\mathcal{I}=-\frac{1}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{s_1s_2\vec{q}^{\,2}\Gamma(D-4)}\left(-\frac{s_1s_2}{s}\right)^{\frac{D}{2}-2}\,. \tag{17}$$

Consequently, using a simple colour algebra and the crossing relations we obtain

$$\mathcal{A}_{AB}^{(8)\ A'GB'}(nf)=\Gamma_{A'A}^{(0)\ i_1}\frac{2s}{t}\Gamma_{B'B}^{(0)\ i_2}gT^c_{i_2i_1}e^*_\mu(k)\left(\frac{p_A^\mu}{p_Ak}-\frac{p_B^\mu}{p_Bk}\right)(-\frac{g^2N}{8(4\pi)^{\frac{D}{2}}})(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2}$$

$$\times\left[3exp\left(-i\pi\left(\frac{D}{2}-2\right)\right)+exp\left(i\pi\left(\frac{D}{2}-2\right)\right)\right]\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{\Gamma(D-4)}\,. \tag{18}$$

The total amplitude is given by the sum of Eqs.(13) and (18).

Assuming the Regge behaviour of the amplitude in the sub-channels s_1 and s_2, from general requirements of analiticity, unitarity and crossing symmetry one has (see Refs. [5, 8])

$$\mathcal{A}_{AB}^{(8)\ A'GB'}=s\Gamma^{i_1}_{A'A}\frac{1}{t_1}T^c_{i_2i_1}\frac{1}{t_2}\Gamma^{i_2}_{B'B}$$
$$\times\left\{\frac{1}{4}\left[\left(\frac{-s_1}{\mu^2}\right)^{\omega_1-\omega_2}+\left(\frac{s_1}{\mu^2}\right)^{\omega_1-\omega_2}\right]\left[\left(\frac{-s}{\mu^2}\right)^{\omega_2}+\left(\frac{s}{\mu^2}\right)^{\omega_2}\right]R\right.$$
$$\left.+\frac{1}{4}\left[\left(\frac{-s_2}{\mu^2}\right)^{\omega_2-\omega_1}+\left(\frac{s_2}{\mu^2}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{-s}{\mu^2}\right)^{\omega_1}+\left(\frac{s}{\mu^2}\right)^{\omega_1}\right]L\right\}\,, \tag{19}$$

where $\omega_i=\omega(t_i)$ and the RRG vertices R and L are real in all physical channels. In the region (5) of small $k_\perp$ this representation reduces to

$$\mathcal{A}_{AB}^{(8)\ A'GB'}=s\Gamma^{i_1}_{A'A}\frac{1}{t}T^c_{i_2i_1}\frac{1}{t}\Gamma^{i_2}_{B'B}\left\{\left[\left(\frac{-s}{\mu^2}\right)^{\omega}+\left(\frac{s}{\mu^2}\right)^{\omega}\right]\frac{R+L}{2}\right.$$

$$\left.+\left[\left(\frac{s}{\mu^2}\right)^{\omega}\left(\ln\left(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\right)-i\pi\right)+\left(\frac{-s}{\mu^2}\right)^{\omega}\ln\left(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\right)\right]\frac{(\omega_1-\omega_2)}{2}\frac{(R-L)}{2}\right\}. \tag{20}$$

Comparing the above form with Eqs. (13) and (18) we conclude that

$$R - L = g e^*_\mu(k) \left(\frac{p_A^\mu}{p_A k} - \frac{p_B^\mu}{p_B k} \right) \frac{t}{\omega_1 - \omega_2}$$

$$\times \left(- \frac{2g^2 N}{(4\pi)^{\frac{D}{2}}} \right) (\vec{k}_\perp^{\,2})^{\frac{D}{2}-2} \frac{\Gamma^2\left(3 - \frac{D}{2}\right)\Gamma^3\left(\frac{D}{2} - 2\right)}{\Gamma(D-4)} \frac{\sin\left(\pi\left(\frac{D}{2} - 2\right)\right)}{\pi} \ ,$$

$$R + L = 2g e^*_\mu(k) \left(\frac{p_A^\mu}{p_A k} - \frac{p_B^\mu}{p_B k} \right) t \left\{ 1 - \omega(t) \ln\left(\frac{-t}{\mu^2}\right) - \frac{g^2 N}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma^2\left(3 - \frac{D}{2}\right)\Gamma^3\left(\frac{D}{2} - 2\right)}{2\Gamma(D-4)} \right.$$

$$\left. \times (\vec{k}_\perp^{\,2})^{\frac{D}{2}-2} \left[\cos\left(\pi\left(\frac{D}{2} - 2\right)\right) - \frac{\sin\left(\pi\left(\frac{D}{2} - 2\right)\right)}{\pi} \ln\left(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\right) \right] \right\} \ . \qquad (21)$$

At $D \to 4$ the above expressions for the vertices coincide, taking into account the charge renormalization, with the small $k_\perp$ limit of the corresponding expressions of Ref. [5] (see Eq.(86) there). Independently we have performed the straightforward calculation of the RRG vertices at small $k_\perp$ for arbitrary D and have obtained the result (21).

In conclusion we stress that although the Gribov's theorem about the soft emission factorization cannot be literally applied in the case of massless theories, it can simplify calculations considerably.

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